NAG C Library Function Document

nag_zgetrs (f07asc)

1 Purpose

nag_zgetrs (f07asc) solves a complex system of linear equations with multiple right-hand sides, $AX = B$, $A^TX = B$ or $A^HX = B$, where A has been factorized by nag_zgetrf (f07arc).

2 Specification

```
void nag_zgetrs (Nag_OrderType order, Nag_TransType trans, Integer n, Integer nrhs,
      const Complex a[], Integer pda, const Integer ipiv[], Complex b[],
      Integer pdb, NagError *fail)
```

3 Description

To solve a complex system of linear equations $AX = B$, $A^TX = B$ or $A^HX = B$, this function must be preceded by a call to nag_zgetrf (f07arc) which computes the LU factorization of A as $A = PLU$. The solution is computed by forward and backward substitution.

If **trans** = **Nag_NoTrans**, the solution is computed by solving $PLY = B$ and then $UX = Y$.

If **trans** = **Nag_Trans**, the solution is computed by solving $U^TY = B$ and then $L^TP^TX = Y$.

If **trans** = **Nag_ConjTrans**, the solution is computed by solving $U^HY = B$ and then $L^HP^TX = Y$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: **order** – Nag_OrderType *Input*

On entry: the **order** parameter specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = **Nag_RowMajor**. See Section 2.2.1.4 of the Essential Introduction for a more detailed explanation of the use of this parameter.

Constraint: **order** = **Nag_RowMajor** or **Nag_ColMajor**.

2: **trans** – Nag_TransType *Input*

On entry: indicates the form of the equations as follows:

if **trans** = **Nag_NoTrans**, $AX = B$ is solved for X;

if **trans** = **Nag_Trans**, $A^TX = B$ is solved for X;

if **trans** = **Nag_ConjTrans**, $A^HX = B$ is solved for X.

Constraint: **trans** = **Nag_NoTrans**, **Nag_Trans** or **Nag_ConjTrans**.

3: **n** – Integer *Input*

On entry: n, the order of the matrix A.

Constraint: **n** ≥ 0.

4: **nrhs** – Integer *Input*

On entry: r, the number of right-hand sides.

Constraint: **nrhs** ≥ 0.

5: **a**[dim] – const Complex *Input*

Note: the dimension, dim, of the array **a** must be at least $\max(1, \textbf{pda} \times \textbf{n})$.

If **order** = **Nag_ColMajor**, the (i, j)th element of the matrix A is stored in **a**$[(j-1) \times \textbf{pda} + i - 1]$ and if **order** = **Nag_RowMajor**, the (i, j)th element of the matrix A is stored in **a**$[(i-1) \times \textbf{pda} + j - 1]$.

On entry: the LU factorization of A, as returned by nag_zgetrf (f07arc).

6: **pda** – Integer *Input*

On entry: the stride separating matrix row or column elements (depending on the value of **order**) in the array **a**.

Constraint: **pda** $\geq \max(1, \textbf{n})$.

7: **ipiv**[dim] – const Integer *Input*

Note: the dimension, dim, of the array **ipiv** must be at least $\max(1, \textbf{n})$.

On entry: the pivot indices, as returned by nag_zgetrf (f07arc).

8: **b**[dim] – Complex *Input/Output*

Note: the dimension, dim, of the array **b** must be at least $\max(1, \textbf{pdb} \times \textbf{nrhs})$ when **order** = **Nag_ColMajor** and at least $\max(1, \textbf{pdb} \times \textbf{n})$ when **order** = **Nag_RowMajor**.

If **order** = **Nag_ColMajor**, the (i, j)th element of the matrix B is stored in **b**$[(j-1) \times \textbf{pdb} + i - 1]$ and if **order** = **Nag_RowMajor**, the (i, j)th element of the matrix B is stored in **b**$[(i-1) \times \textbf{pdb} + j - 1]$.

On entry: the n by r right-hand side matrix B.

On exit: the n by r solution matrix X.

9: **pdb** – Integer *Input*

On entry: the stride separating matrix row or column elements (depending on the value of **order**) in the array **b**.

Constraints:

if **order** = **Nag_ColMajor**, **pdb** $\geq \max(1, \textbf{n})$;
if **order** = **Nag_RowMajor**, **pdb** $\geq \max(1, \textbf{nrhs})$.

10: **fail** – NagError * *Output*

The NAG error parameter (see the Essential Introduction).

6 Error Indicators and Warnings

NE_INT

On entry, **n** = ⟨*value*⟩.
Constraint: **n** ≥ 0.

On entry, **nrhs** = ⟨*value*⟩.
Constraint: **nrhs** ≥ 0.

On entry, **pda** = ⟨*value*⟩.
Constraint: **pda** > 0.

On entry, **pdb** = ⟨*value*⟩.
Constraint: **pdb** > 0.

NE_INT_2

On entry, **pda** $= \langle value \rangle$, **n** $= \langle value \rangle$.
Constraint: **pda** $\geq \max(1, \mathbf{n})$.

On entry, **pdb** $= \langle value \rangle$, **n** $= \langle value \rangle$.
Constraint: **pdb** $\geq \max(1, \mathbf{n})$.

On entry, **pdb** $= \langle value \rangle$, **nrhs** $= \langle value \rangle$.
Constraint: **pdb** $\geq \max(1, \mathbf{nrhs})$.

NE_ALLOC_FAIL

Memory allocation failed.

NE_BAD_PARAM

On entry, parameter $\langle value \rangle$ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \leq c(n)\epsilon P|L|\,|U|,$$

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x - \hat{x}\|_\infty}{\|x\|_\infty} \leq c(n)\,\mathrm{cond}(A, x)\epsilon$$

where $\mathrm{cond}(A, x) = \||A^{-1}|\,|A|\,|x|\|_\infty / \|x\|_\infty \leq \mathrm{cond}(A) = \||A^{-1}|\,|A|\|_\infty \leq \kappa_\infty(A)$. Note that $\mathrm{cond}(A, x)$ can be much smaller than $\mathrm{cond}(A)$, and $\mathrm{cond}(A^H)$ (which is the same as $\mathrm{cond}(A^T)$) can be much larger (or smaller) than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling nag_zgerfs (f07avc), and an estimate for $\kappa_\infty(A)$ can be obtained by calling nag_zgecon (f07auc) with **norm** $=$ **Nag_InfNorm**.

8 Further Comments

The total number of real floating-point operations is approximately $8n^2r$.

This function may be followed by a call to nag_zgerfs (f07avc) to refine the solution and return an error estimate.

The real analogue of this function is nag_dgetrs (f07aec).

9 Example

To solve the system of equations $AX = B$, where

$$A = \begin{pmatrix} -1.34 + 2.55i & 0.28 + 3.17i & -6.39 - 2.20i & 0.72 - 0.92i \\ -0.17 - 1.41i & 3.31 - 0.15i & -0.15 + 1.34i & 1.29 + 1.38i \\ -3.29 - 2.39i & -1.91 + 4.42i & -0.14 - 1.35i & 1.72 + 1.35i \\ 2.41 + 0.39i & -0.56 + 1.47i & -0.83 - 0.69i & -1.96 + 0.67i \end{pmatrix}$$

and

$$B = \begin{pmatrix} 26.26 + 51.78i & 31.32 - \ \ 6.70i \\ 6.43 - \ \ 8.68i & 15.86 - \ \ 1.42i \\ -5.75 + 25.31i & -2.15 + 30.19i \\ 1.16 + \ \ 2.57i & -2.56 + \ \ 7.55i \end{pmatrix}.$$

Here A is nonsymmetric and must first be factorized by nag_zgetrf (f07arc).

9.1 Program Text

```
/* nag_zgetrs (f07asc) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagx04.h>

int main(void)
{
  /* Scalars */
  Integer  i, ipiv_len, j, n, nrhs, pda, pdb;
  Integer  exit_status=0;
  NagError fail;
  Nag_OrderType order;
  /* Arrays */
  Complex  *a=0, *b=0;
  Integer  *ipiv=0;

#ifdef NAG_COLUMN_MAJOR
#define A(I,J) a[(J-1)*pda + I - 1]
#define B(I,J) b[(J-1)*pdb + I - 1]
  order = Nag_ColMajor;
#else
#define A(I,J) a[(I-1)*pda + J - 1]
#define B(I,J) b[(I-1)*pdb + J - 1]
  order = Nag_RowMajor;
#endif

  INIT_FAIL(fail);
  Vprintf("f07asc Example Program Results\n\n");
  /* Skip heading in data file */
  Vscanf("%*[^\n] ");
  Vscanf("%ld%ld%*[^\n] ", &n, &nrhs);
#ifdef NAG_COLUMN_MAJOR
  pda = n;
  pdb = n;
#else
  pda = n;
  pdb = nrhs;
#endif
  ipiv_len = n;

  /* Allocate memory */
  if ( !(a = NAG_ALLOC(n * n, Complex)) ||
       !(b = NAG_ALLOC(n * nrhs, Complex)) ||
       !(ipiv = NAG_ALLOC(ipiv_len, Integer)) )
    {
      Vprintf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }

  /* Read A and B from data file */
```

```
  for (i = 1; i <= n; ++i)
    {
      for (j = 1; j <= n; ++j)
        {
          Vscanf(" ( %lf , %lf )", &A(i,j).re, &A(i,j).im);

        }
    }
  Vscanf("%*[^\n] ");
  for (i = 1; i <= n; ++i)
    {
      for (j = 1; j <= nrhs; ++j)
        Vscanf(" ( %lf , %lf )", &B(i,j).re, &B(i,j).im);
    }
  Vscanf("%*[^\n] ");

  /* Factorize A */
  f07arc(order, n, n, a, pda, ipiv, &fail);
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from f07arc.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }

  /* Compute solution */
  f07asc(order, Nag_NoTrans, n, nrhs, a, pda, ipiv, b, pdb, &fail);
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from f07asc.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }

  /* Print solution */
  x04dbc(order, Nag_GeneralMatrix, Nag_NonUnitDiag, n, nrhs, b, pdb,
         Nag_BracketForm, "%7.4f", "Solution(s)", Nag_IntegerLabels, 0,
         Nag_IntegerLabels, 0, 80, 0, 0, &fail);
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from x04dbc.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
 END:
  if (a) NAG_FREE(a);
  if (b) NAG_FREE(b);
  if (ipiv) NAG_FREE(ipiv);
  return exit_status;
}
```

9.2 Program Data

```
f07asc Example Program Data
  4  2                                                      :Values of N and NRHS
 (-1.34, 2.55) ( 0.28, 3.17) (-6.39,-2.20) ( 0.72,-0.92)
 (-0.17,-1.41) ( 3.31,-0.15) (-0.15, 1.34) ( 1.29, 1.38)
 (-3.29,-2.39) (-1.91, 4.42) (-0.14,-1.35) ( 1.72, 1.35)
 ( 2.41, 0.39) (-0.56, 1.47) (-0.83,-0.69) (-1.96, 0.67)  :End of matrix A
 (26.26, 51.78) (31.32, -6.70)
 ( 6.43, -8.68) (15.86, -1.42)
 (-5.75, 25.31) (-2.15, 30.19)
 ( 1.16,  2.57) (-2.56,  7.55)                              :End of matrix B
```

9.3 Program Results

```
f07asc Example Program Results

 Solution(s)
                    1                   2
 1  ( 1.0000, 1.0000)  (-1.0000,-2.0000)
 2  ( 2.0000,-3.0000)  ( 5.0000, 1.0000)
 3  (-4.0000,-5.0000)  (-3.0000, 4.0000)
 4  ( 0.0000, 6.0000)  ( 2.0000,-3.0000)
```